ELIZADE UNIVERSITY, ILARA-MOKIN, ONDO STATE.

FACULTY OF ENGINEERING
Department of Electrical and Computer Engineering
First Semester 2017/2018 Session Examination

COURSE TITLE: Digital Signal Processing

COURSE CODE: ECE 519

COURSE LECTURER: DR. A. M. JUBRIL

TIME ALLOWED: 3 HOURS

INSTRUCTIONS:

1. ANSWER FOUR QUESTIONS ONLY
2. SEVERE PENALTIES APPLY FOR MISCONDUCT, CHEATING, POSSESSION OF UNAUTHORIZED MATERIALS DURING EXAM
3. YOU ARE NOT ALLOWED TO BORROW ANY WRITING MATERIALS DURING THE EXAMINATION

HODs SIGNATURE: ——————————

1. (a) Consider the following length-7 sequences defined for $-3 \le n \le 3$: $x[n]$={3, -2, 0, 1, 4, 5, 2}, $y[n]$={0, 7, 1, -3, 4, 9, -2}, $w[n]$={-5, 4, 3, 6, -5, 0, 1}. Generate the following sequences:

   i. $u[n] = x[n] + y[n]$ (2 marks)

   ii. $v[n] = x[n] \cdot w[n]$ (2 marks)

   iii. $r[n] = 3.2\, x[n]$ (2 marks)

   (b) Determine the even and the odd parts of the sequences $x[n]$, $y[n]$ and $w[n]$ in 1.(a). (4 marks)

   (c) Analyze the block diagram of the system in Figure 1 and develop a relation between $x[n]$ and $y[n]$. (3 marks)

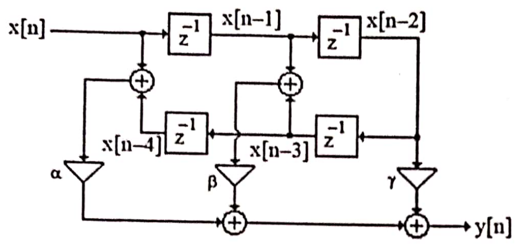

Figure 1:

   (d) Show that a causal real sequence $x[n]$ can be fully recovered from its even part $x_{ev}[n]$ for all $n \ge 0$, whereas it can be recovered from its odd part $x_{od}[n]$ for all $n > 0$. (4 marks)

   (e) Determine the boundedness of the sequences $x[n] = A\alpha^n \mu[n]$ and $y[n] = 5\cos^3(\omega_o n^2)$, where A and α are complex numbers, and $|\alpha| < 1$ (3 marks)

2. (a) Let $x_{ev}[n]$ and $x_{od}[n]$ denote, respectively,the even and odd parts of a square-summable $x[n]$. Prove the following result:

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_{ev}^2[n] + \sum_{n=-\infty}^{\infty} x_{od}^2[n]$$

   (4 marks)

   (b) Compute the energy of the length-N sequence

$$x[n] = \cos\left(\frac{2\pi kn}{N}\right)$$

   (4 marks)

   (c) The sequence of Fibonacci numbers $f[n]$ is a causal sequence defined by

$$f[n] = f[n-1] + f[n-2], \qquad n \ge 2$$

   with $f[0] = 0$ and $f[1] = 1$.

i. Develop an exact formula to calculate $f[n]$ directly for any n. (4 marks)

ii. Show that $f[n]$ is the impulse response of a causal linear time-invariant system described by the difference equation

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

(4 marks)

(d) Determine the expression for the impulse response of the factor-of-3 linear interpolator given as

$$y[n] = x[n] + \frac{1}{3}(x[n-1] - x[n+2]) + \frac{2}{3}(x[n-2] - x[n+1])$$

(4 marks)

3. (a) Determine the DTFT of the causal sequence

$$A\alpha^n \cos(\omega_o n + \phi)\mu[n]$$

(5 marks)

(b) Determine the expression for the impulse response of the linear time-invariant system in Figure 2. (4 marks)

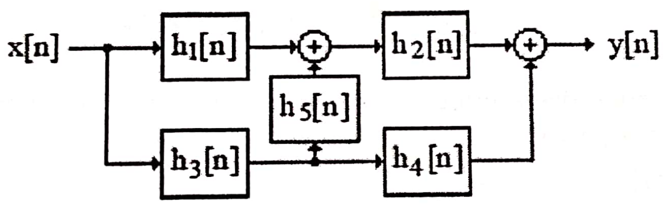

Figure 2:

(c) Let $X(e^{j\omega})$ denotes the DTFT of a real sequence $x[n]$. Determine the inverse DTFT of the following in terms of $x[n]$:

i. $X_{re}(e^{j\omega})$ (2 marks)

ii. $jX_{im}(e^{j\omega})$ (2 marks)

(d) Determine the DTFT of each of the following sequences:

i. $x_1[n] = \alpha^n \mu[n-1], \quad |\alpha| < 1$ (2 marks)

ii. $x_2[n] = \alpha^n \mu[n], \quad |\alpha| < 1$ (2 marks)

iii. $x_3[n] = \begin{cases} 1, & -N \leq n \leq N; \\ 0, & \text{otherwise.} \end{cases}$ (2 marks)

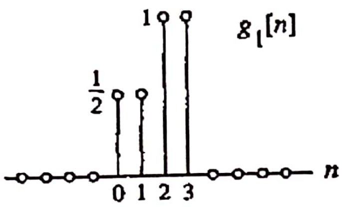

Figure 3:

4. (a) Let $G_1(e^{j\omega})$ denotes the discrete-time Fourier transform (DTFT) of the sequence $g_1[n]$ as shown in Figure 3. Evaluate $G_1(e^{j\omega})$. (5 marks)

   (b) The discrete Fourier transform (DFT) pair is

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{\frac{-j2\pi nk}{N}}$$

   and

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \cdot e^{\frac{-j2\pi nk}{N}} s$$

   Determine the discrete Fourier series coefficient of the following periodic sequences:

   i. $x_1[n] = \cos(\pi n/4)$ (4.5 marks)

   ii. $x_2[n] = \sin(\pi n/3) + \cos(\pi n/4)$ (4.5 marks)

   (c) Let $x[n]$, $0 \leq n \leq N-1$, be a length-N real sequence with an N-point DFT $X[k]$, $0 \leq k \leq N-1$.

   i. Show that $X[N-k] = X^*[k]$. (2 marks)

   ii. Show that $X[0]$ is real. (2 marks)

   iii. If N is even, show that $X[N/2]$ is real. (2 marks)

5. (a) Let $g[n]$ and $h[n]$ be two finite-length sequences given as

$$\{g[n]\} = \{-3, 2, 4\}, \qquad \{h[n]\} = \{2, -4, 0, 1\}$$

   i. Determine $y[n] = \sum_{n=-\infty}^{\infty} h[n-k]g[k]$ (3 marks)

   ii. Extend $g[n]$ to a length-4 sequence $g_e[n]$ by zero-padding and compute $y_e[n] = \sum_{n=-\infty}^{\infty} h[n-k]g_e[k]$ (2 marks)

   (b) Consider up-sampling the sequence $x[n]$ by an integer factor $L > 1$ to give a new sequence $y[n]$, which implies that $L-1$ equidistance zero-valued samples are inserted between each consecutive samples of $x[n]$,

$$y[n] = \begin{cases} x[n/L], & n = 0, L, 2L, \ldots; \\ 0, & \text{otherwise.} \end{cases}$$

If the z-transform of $x[n]$ is denoted as $X(z)$, express $Y(z)$ in terms of $X(z)$. (4 marks)

(c) Let $X(z)$ denotes the z-transform of $x[n] = (0.4)^n \mu[n]$. Determine the inverse z-transform of $X(z^2)$. (5 marks)

(d) A finite impulse response (FIR) linear time-invariant discrete-time is described by the difference

$$y[n] = a_1 x[n+k] + a_2 x[n+k-1] + a_3 x[n+k-2] + a_2 x[n+k-3] + a_1 x[n+k-4]$$

where $y[n]$ and $x[n]$ denotes, respectively, the output and the input sequence. Determine the expression for its frequency response $H(e^{j\omega})$. For what values of the constant k will the system have a frequency response $H(e^{j\omega})$ that is real function of ω. (6 marks)

6. (a) Determine a closed-form expression for the frequency response $H(e^{j\omega})$ of the LTI discrete-time system characterized by an impulse response

$$h[n] = \delta[n] - \alpha\delta[n - n_o]$$

where $|\alpha| < 1$. What are the maximum and the minimum of its magnitude response. (7 marks)

(b) Determine a closed-form expression for the frequency response $H(e^{j\omega})$ of the LTI discrete-time system characterized by an impulse response

$$h[n] = g[n] * g[n] * g[n]$$

where $g[n] = \delta[n] - \alpha\delta[n - n_o]$ (5 marks)

(c) Determine the inverse discrete-time Fourier transform (DTFT) of the DTFT given as

$$H(e^{j\omega}) = 1 + 2\cos\omega + 3\cos 2\omega$$

(8 marks)

Property	Sequence	z -Transform	ROC
	$g[n]$ $h[n]$	$G(z)$ $H(z)$	$\mathcal{R}_g$ $\mathcal{R}_h$
Conjugation	$g^*[n]$	$G^*(z^*)$	$\mathcal{R}_g$
Time-reversal	$g[-n]$	$G(1/z)$	$1/\mathcal{R}_g$
Linearity	$\alpha g[n] + \beta h[n]$	$\alpha G(z) + \beta H(z)$	Includes $\mathcal{R}_g \cap \mathcal{R}_h$
Time-shifting	$g[n - n_o]$	$z^{-n_o} G(z)$	$\mathcal{R}_g$, except possibly the point $z = 0$ or ∞
Multiplication by an exponential sequence	$\alpha^n g[n]$	$G(z/\alpha)$	$\lvert\alpha\rvert \mathcal{R}_g$
Differentiation of $G(z)$	$n g[n]$	$-z \dfrac{dG(z)}{dz}$	$\mathcal{R}_g$, except possibly the point $z = 0$ or ∞
Convolution	$g[n] \circledast h[n]$	$G(z)H(z)$	Includes $\mathcal{R}_g \cap \mathcal{R}_h$
Modulation	$g[n]h[n]$	$\frac{1}{2\pi j} \oint_C G(v) H(z/v) v^{-1}\, dv$	Includes $\mathcal{R}_g \mathcal{R}_h$
Parseval's relation		$\sum_{n=-\infty}^{\infty} g[n]h^*[n] = \frac{1}{2\pi j} \oint_C G(v) H^*(1/v^*) v^{-1}\, dv$	

Note: If $\mathcal{R}_g$ denotes the region $R_{g-} < |z| < R_{g+}$ and $\mathcal{R}_h$ denotes the region $R_{h-} < |z| < R_{h+}$, then $1/\mathcal{R}_g$ denotes the region $1/R_{g+} < |z| < 1/R_{g-}$ and $\mathcal{R}_g \mathcal{R}_h$ denotes the region $R_{g-} R_{h-} < |z| < R_{g+} R_{h+}$.

Table 3.2: General properties of the discrete-time Fourier transform of sequences.

Type of Property	Sequence	Discrete-Time Fourier Transform
	$g[n]$ $h[n]$	$G(e^{j\omega})$ $H(e^{j\omega})$
Linearity	$\alpha g[n] + \beta h[n]$	$\alpha G(e^{j\omega}) + \beta H(e^{j\omega})$
Time-shifting	$g[n - n_o]$	$e^{-j\omega n_o} G(e^{j\omega})$
Frequency-shifting	$e^{j\omega_o n} g[n]$	$G\left(e^{j(\omega-\omega_o)}\right)$
Differentiation in frequency	$n g[n]$	$j \dfrac{dG(e^{j\omega})}{d\omega}$
Convolution	$g[n] \circledast h[n]$	$G(e^{j\omega}) H(e^{j\omega})$
Modulation	$g[n]h[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} G(e^{j\theta}) H(e^{j(\omega-\theta)})\, d\theta$
Parseval's relation	$\sum_{n=-\infty}^{\infty} g[n]h^*[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} G(e^{j\omega}) H^*(e^{j\omega})\, d\omega$	